The Spotting Distribution of Wildland Fires

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Spotting

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Examples of spotting

- Fort McMurray fire
- Slave Lake fire
- California
- Australia
- Spain

Research Questions

1. What is a spotting distribution?
2. What role does spotting play in fire spread ?
3. What is the probability that a fire breaches an obstacle?

Spotting model

(Hillen, Greese, Martin, de Vries, JBD 2014)

$u(x, t)$ fire probability
$v(x, t)$ available fuel

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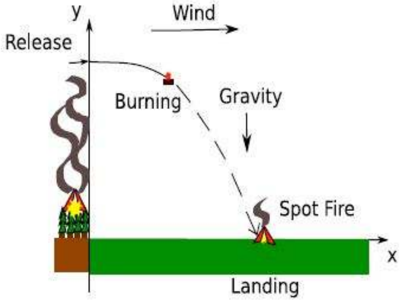
$$
\begin{aligned}
u_t &= \underbrace{Du_{xx}}_{\text{local spread}} + \underbrace{\int S(x,y,u(x,t))u(y,t)dy}_{\text{spotting}} \\
&\quad + \underbrace{\gamma c(u,v)v}_{\text{combustion}} - \underbrace{\delta(u)u}_{\text{extinciton}} \\
v_t &= \underbrace{-c(u,v)v}_{\text{fuel consumption}}
\end{aligned}
$$

Spotting distribution

How to find the spotting distribution $S(x, y, u)$?

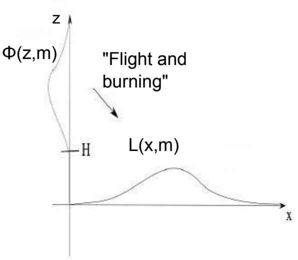
Major components

1. launching: launching distribution $\phi(z, m)$, z: height, m: mass.
2. horizontal wind: $w(z) > 0$.
3. terminal falling velocity $v(t, m) < 0$.
4. burning rate during flight: $f(t, z, m) < 0$.
5. ignition upon landing: $E(m)$.



Goal

$$\begin{pmatrix} \text{launching} \\ \text{distribution} \\ \phi(z,m) \end{pmatrix} \mapsto \begin{pmatrix} \text{landing} \\ \text{distribution} \\ \mathbb{L}(x,m) \end{pmatrix}$$



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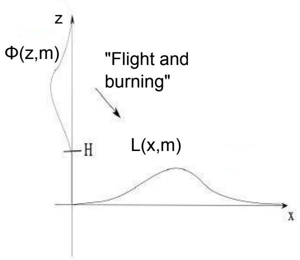

- Total landed mass:

$$M(x) := \int_0^{\bar{m}} m\mathbb{L}(x,m)dm$$

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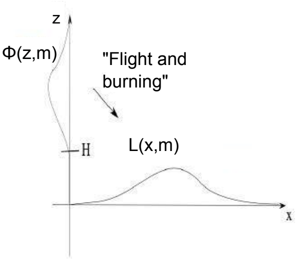

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- Spotting distribution

$$\mathbb{S}(x) = E(M(x)).$$

Flying fire brands

$p(t, x, z, m)$: fire brands at time $t \geq 0$, location $x \geq 0$, height $z \geq 0$ and mass $m \geq 0$.

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$$\frac{\partial p}{\partial t} + \frac{\partial w(z)p}{\partial x} + \frac{\partial v(t, m)p}{\partial z} + \frac{\partial f(t, z, m)p}{\partial m} = 0$$

$w(z)$: wind velocity
$v(t, m)$: terminal falling velocity
$f(t, z, m)$: combustion

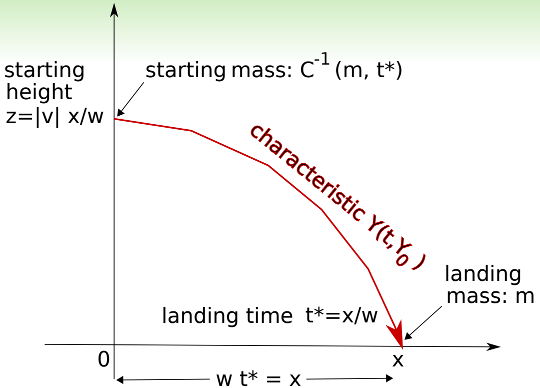
The Confetti Problem

Initial conditions, point release:

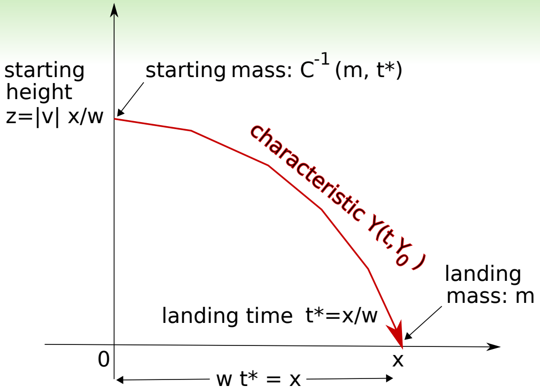
$$p(0, x, z, m) = \begin{cases} N\phi(z, m) & \text{for} \quad x = 0, t = 0 \\ 0 & \text{else} \end{cases}$$

Landing distribution

$$\mathbb{L}(x, m) = \lim_{t \to \infty} \int_0^t p(s, x, 0, m) ds$$



- Let $C(t, m)$ denote the combustion operator.
- A fire brand of mass m landing at x flew for a time $t^*(x)$ and has burned from an initial mass $C^{-1}(m, t^*(x))$.



- Let $C(t, m)$ denote the combustion operator.
- A fire brand of mass m landing at x flew for a time $t^*(x)$ and has burned from an initial mass $C^{-1}(m, t^*(x))$.
- and the spotting distribution is

$$\mathbb{S}(x) = E\left(\int_0^{\bar{m}} m\mathbb{L}\Big(x, C^{-1}(m, t^*(x)\Big)\ dm\right),$$

where $E(M)$ denotes the ignition probability of a landed burning mass M.

Physics

Based on physical details:

Physical process	case 1	case 2	case 3
Launching	unique height	normally distributed	heights and masses independent
Mass distribution	normal	power law	slash burning
Wind	constant	logarithmic	power law
Falling	constant	cylindrical firebrands	mass dependent
Combustion	constant burn rate Fernandez-Perro	Tarifa's model Albini's model	no combustion
Ignition	piecewise linear	heaviside step function	smooth step function
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(A total of 1400 cases !)

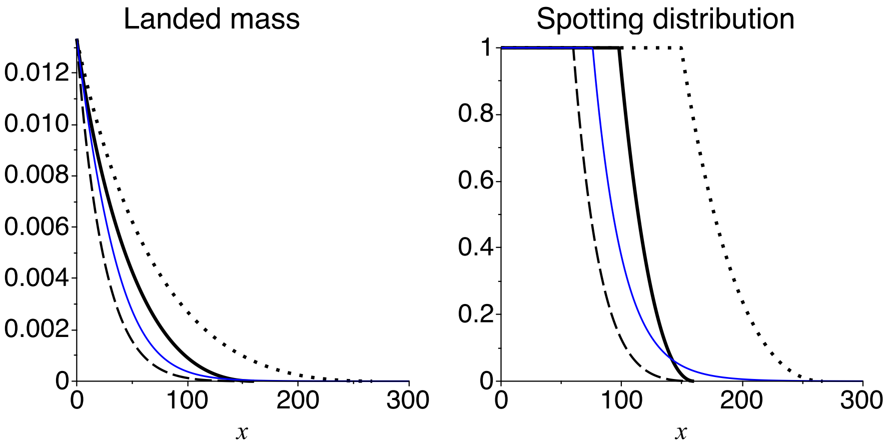
Examples

- Constant wind $w > 0$
- Constant falling velocity $v < 0$
- Maximum loftable mass $\bar{m} = 0.004(kg)$
- Minimum mass to ignite a fire $\underline{m} = 0.001$
- Manzello, power-law mass distribution: $\mu(m) = am^{-0.5}$
- Constant burn rate $-\kappa$, or Tarifa's model.
- Number of fire brands N
- Hight distribution $Z(z) = \lambda e^{-\lambda z}$

	w	v	$\bar{m}$	$\underline{m}$	a	κ	N	λ	x_{max}
base case (solid)	2	-1	0.004	0.001	7.91	0.00005	1000	0.01	160
slow burner (dotted)	2	-1	0.004	0.001	7.91	0.00003	1000	0.01	266.66
lower release height (dashed)	2	-1	0.004	0.001	7.91	0.00005	1000	0.05	160

	w	v	$\bar{m}$	$\underline{m}$	a	η	N	λ	x_{max}
Tarifa's case (thin blue)	2	-1	0.004	0.001	7.91	0.000286	1000	0.01	∞

base case (solid)
slow burner (dotted)
lower release height (dashed)
Tarifa's case (thin blue)



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Thank you